HEAT TRANSFER AND INSULATION IN VACUUM FURNACES

Heat transfer calculations are important in furnace design. Computers can make the work easy, but knowing how to do the calculations by hand gives a better feeling for the problem and provides an appreciation of what actually is happening.

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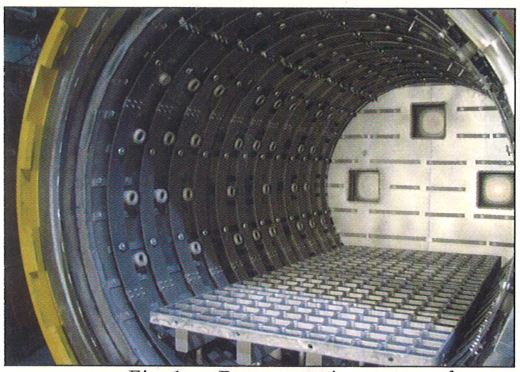

Fig. 1 — Representative vacuum furnace hot zone with tray for carrying workload.

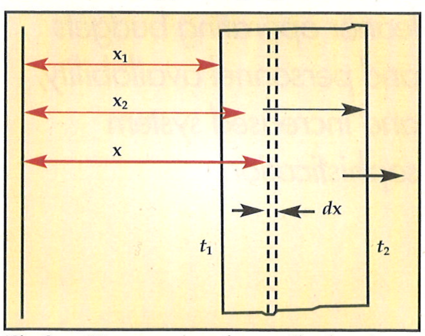

Fig. 2 — Homogeneous plane wall, where $t_2 < t_1$.

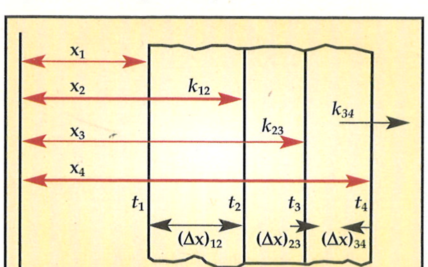

Fig. 3 — Composite plane wall.

Heat transfer is important in many facets of engineering, and is especially true in the design of furnaces. In particular, to establish power requirements for a given furnace application, you must determine both the amount of energy input to heat the load and insulation and the heat loss due to the energy transmitted through the insulation. This review considers only the heat transfer through the insulation. Figure 1 shows a representative hot zone in a vacuum furnace. Factors such as specific heat and weight of the load, as well as thermal conductivity and thickness of the insulating material must be determined to calculate heat-transfer to establish the power supply requirements.

Three types of heat transfer are conduction, convection, and radiation. The case of the heat transfer through the insulation basically involves steady-state conduction, which simplifies the calculations. Four simple cases considered in this introductory review are:

- Conduction through a homogeneous plane wall
- Conduction through a composite plane wall
- Conduction through a homogeneous cylindrical wall
- Conduction through a composite cylindrical wall

The fundamental relation for the steady flow of heat by conduction is given by the expression:

$$Q = kA\,(dt/dx) \qquad [1]$$

where Q is heat flow (Btu/h), k is thermal conductivity (Btu/h ft °F), A is area (ft^2), t is temperature (°F), and x is thickness (ft). (Note that in many tables, thermal conductivity is given in terms of inches, which requires making the appropriate dimensional changes in the other quantities.)

Case 1: Conduction through a homogeneous plane wall

Figure 2 shows the model for this case, where it is assumed that the homogeneous wall material has a thermal conductivity that does not vary with temperature. The heat flow through a wall of thickness $(x_2\text{-}x_1)$ having a temperature differential of $(t_1\text{-}t_2)$ can be calculated from equation [1], giving:

$$Q = kA\,(\Delta t/\Delta x) \qquad [2]$$

or

$$Q = kA\,\{(t_1\text{-}t_2)/(x_2\text{-}x_1)\} \qquad [3]$$

Case 2: Conduction through a composite plane wall

Figure 3 shows the model for this case, and heat flow is given by:

$$Q = A\,\{\Sigma\,\Delta t\,/\Sigma(\Delta x/k)\} \qquad [4]$$

or

$$Q = \frac{A\,(t_1 - t_4)}{\dfrac{\Delta x_{12}}{k_{12}} + \dfrac{\Delta x_{23}}{k_{23}} + \dfrac{\Delta x_{34}}{k_{34}}} \qquad [5]$$

Equation [5] also can be used to determine the heat flow through layers of the same material, where the thermal conductivity varies with temperature. If the problem requires more than the three layers shown in Figure 3, then an additional term is added to Equation [5] for each additional layer.

Case 3: Conduction through a homogeneous cylindrical wall

Figure 4 shows the model for this condition, and heat flow is given by:

$$Q' = \frac{2\,\Pi\,k\,(t_1 - t_2)}{\ln\,(r_2/r_1)} \qquad [6]$$

where Q′ is the rate of heat flow per unit length of the cylindrical insulation (Btu/hr ft). It is assumed in Equation [6] that the thermal conductivity of the homogeneous cylindrical insulation does not vary with temperature.

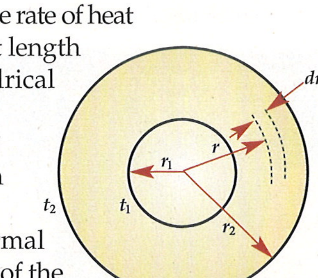

Fig. 4 — Homogeneous cylindrical wall.

Case 4: Conduction through a composite cylindrical wall

Figure 5 shows the model for composite cylindrical wall of insulation. If we consider that the heat flow through each layer is similar to that given by Equation [6], a summation of the layers gives the following solution:

$$Q' = \frac{2\Pi (t_1 - t_m)}{\sum \dfrac{\ln \{(r_{(m+1)}) / r_m\}}{k}} \qquad [7]$$

where: $\sum$ from m = 1 to m = n-1

For the case shown in Figure 5, Equation [7] reduces to:

$$Q' = \frac{2\Pi (t_1 - t_2)}{\dfrac{\ln (r_2/r_1)}{k_{12}} + \dfrac{\ln (r_3/r_2)}{k_{23}}} \qquad [8]$$

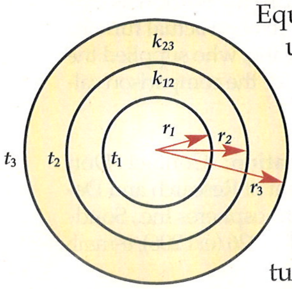

Fig. 5 — Composite cylindrical wall.

Equation [8] can be used for multiple layers of the same insulating material where the conductivity varies with temperature. The approach in that case would be to assign the appropriate value of k to a particular layer associated with the mean temperature of that layer.

The use of these calculations to determine insulation requirements is illustrated in the following practical situation.

Problem: Calculate the heat flow through the insulation in a vacuum furnace under a full atmosphere (760 torr) of nitrogen, where there are 10 layers of graphite felt insulation, each layer being 1 in. thick. The inside surface of the first layer of graphite felt insulation sees a temperature of 4700°F, while the outside of the last layer of graphite has a maximum temperature of 522°F. The insulation is in a cylindrical form with an inside diameter of 84 in., with the same type of layers of insulation at the circular ends.

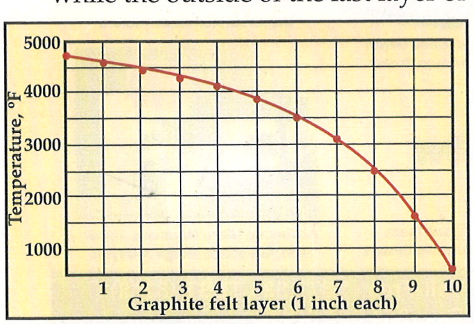

Fig. 6 — Temperature distribution through ten 1-in.thick layers of graphite felt.

Determining the heat flow through the circular end wall of insulation requires the use of Equation [5] rather than the more complicated Equation [8] that addresses the heat transfer through the cylindrical insulating shell. The temperature distribution through the graphite felt will be almost parabolic in nature as shown in Figure 6. The thermal data required for the calculations are shown in Table I. The values for k may be determined from Figure 7.

From Equation [4]

$$Q = \frac{A \sum (\Delta t)}{\sum (\Delta x / k)}$$

In this case, Δx = 1 in., and 1 ft^2 of

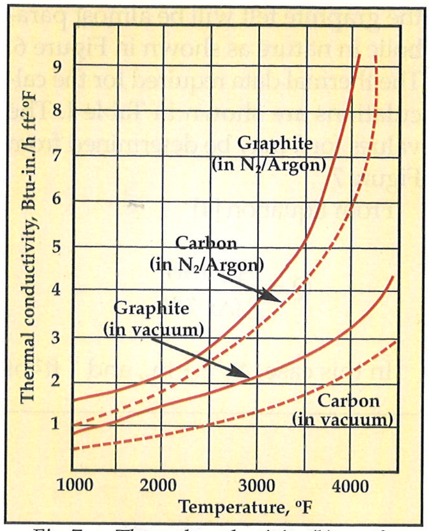

Fig. 7 — Thermal conductivity (k) as a function of temperature for graphite or carbon in a vacuum.

end insulation is examined giving

$$Q = \frac{(1)\,(4178)}{(1.89)} = 2{,}211 \text{ Btu/h ft}^2$$
$$= 2{,}211/144 = 15.35 \text{ Btu/h in.}^2$$
$$= (15.35) \times (0.259) = 3.98 \text{ W/in.}^2$$

The calculation above represents the energy input required at steady state for the heat flow through the insulation, while maintaining the temperature in the furnace. However, the total power input required also must consider the heat storage in the insulation and the heat input to bring the charge up to temperature.

Today, there are computer programs available to perform these calculations that make the job much easier. However, doing the above calculation the long way at least once provides the individual with better appreciation of what actually is going on and a feeling for the problem. One such computer solution (Thermal Ceramics Heat Flow Program) for the above problem provided a value of 3.88 W/in.^{2}.

*Table 1 — Thermal data for heat flow calculations**

Insulating layer	Face temperature, °F	Thermal conductivity (k), Btu in./h ft^2 °F	$\Delta x / k$
1	4700	16	6.25 x 10^{-2}
2	4574	14	7.14 x 10^{-2}
3	4430	12.3	8.13 x 10^{-2}
4	4264	10.4	9.62 x 10^{-2}
5	4067	9.1	10.99 x 10^{-2}
6	3826	7.1	14.08 x 10^{-2}
7	3520	5.1	19.61 x 10^{-2}
8	3109	4.0	25.00 x 10^{-2}
9	2519	2.8	35.71 x 10^{-2}
10	1651	1.9	52.63 x 10^{-2}
Outside	522		Total = 189.16 x 10^{-2}

**Ten 1-in. thick graphite felt layers*

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